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Super Beta Combination Labeling of Graphs

[1] **R.Subramoniam,** [2] **T. Tharma Raj**
[1, 2] Assistant Professor,
[1, 2]Department of Mathematics,
Lekshmipuram College of Arts and Science, Lekshmipuram, Neyyoor - 629802.
[1]Email:trajtr@gmail.com, [2] Email:subs19@gmail.com.

ABSTRACT

Let $G(V,E)$ be a graph with p vertices and q edges. A graph $G(p,q)$ is said to be a Super Beta combination graph if there exist a bijection $f:V(G)\to\{1,2,3,...,p\}$ such that the induced function $f^s:E(G)\to N$, N is a natural number, given by $f^s(uv)=\frac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$, for every edges uv of G and are all distinct and the function f is called the Super Beta combination labeling of G. In this paper, we prove, Star $K_{1,n}$, graph $P_m\cup P_n$, double triangular snake $D(T_n)$, graph $T_n\Theta K_1$, graph $Q_n\Theta K_1$, graph $T_m\cup Q_n$, graph $(P_m\Theta K_1)\cup(C_n\Theta K_1)$ are the Super Beta combination graphs.

Key Words: Super Beta combination graph and Super Beta combination labeling.
Mathematical subject classification (2010) 05C78.

1. INTRODUCTION

The terminology and notations used here are in the sense of Harary [5]. *Graph labelings* were first introduced in the mid sixties. Graph labeling, where the *vertices* and *edges* are assigned real values or subsets of a set are subject to certain conditions. The concepts of graph labeling have been applied in missile guidance codes, Radar location codes, coding theory, circuit design, communication network, X-ray crystallography and radio-astronomy, etc. Throughout this paper, by a graph we mean a finite, undirected, simple graph. The vertex set and the edge set of a graph G are denoted by V(G) and E(G) respectively. Let G(p,q) be a graph with p = | V(G) | vertices and q = | E(G) | edges. A detailed survey of graph labeling can be found in J.A.Gallian survey [4].Combinations play a major role in combinatorial problems. S.M.Hegde and Sudhakar Shetty introduced the concept of combinatorial labeling of graphs [6]. Motivated by these works, we defined Beta combination labeling and Super Beta combination labeling [9,10,11]. For detailed study of various graph labeling, we refer [7, 8]. In this paper we investigate a few Super Beta combination graphs. We use the following definitions in the subsequent sections.

Definition 1.1 [11] A graph $G(p,q)$ is said to be a Super Beta combination graph if there exist a bijection $f:V(G)\to\{1,2,3,...,p\}$ such that the induced function $f^s:E(G)\to N$, N is a natural number, given by $f^s(uv)=\frac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$, for every edges uv of G are all distinct and the function f is called the Super Beta combination labeling of G.

Definition 1.2.[5] A walk in G is a finite non-null sequence $w=v_0e_1v_1e_2v_2....e_kv_k$ whose terms are alternatively vertices and edges, such that, for $1\le i\le k$, the ends of e_i are v_{i-1} and v_i. A walk is said be a path if all the vertices are distinct. It is closed if $v_0 = v_k$ and is open otherwise. A closed path with $n \geq 3$ is called a cycle.

Definition 1.3.[4] A triangular snake is obtained from a path $v_1, v_2, \ldots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i=1,2,\ldots,n-1$.

Definition 1.4.[4]: A double triangular snake consists of two triangular snakes that have a common path. That is , a double triangular snake is obtained from a path $u_1, u_2, \ldots., u_n$ by joining u_i and u_{i+1} to new vertex v_i for $i = 1,2,\ldots., n-1$ and to a new vertex w_i for $i = 1,2,\ldots., n-1$.

Definition 1.5.[4] A quadrilateral snake is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i , u_{i+1} to new vertices v_i , w_i . That is, every edge of the path is replaced by the cycle.

Definition 1.6.[4] The corona $G_1 \Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p points) and p copies of G_2 and then joining the ith point of G_1 to every point in the ith copy of G_2 .

Definition 1.7. [5] A star S_n is a complete bipartite graph $K_{1,n}$ is a tree with one internal node and n leaves.

Definition 1.8. [5] The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 U\ G_2$ with vertex set $V = V_1 U\ V_2$ and the edge set $E = E_1 U\ E_2$.

. In this paper, we prove, Star $K_{1,n}$, graph $P_m \cup P_n$, double triangular snake $D(T_n)$, graph $T_n \Theta K_1$, graph $Q_n \Theta K_1$, graph $T_m \cup Q_n$, graph $(P_m \Theta K_1) \cup (C_n \Theta K_1)$ are the Super Beta combination graphs.

2.Main Results

Theorem 2.1. Every star $K_{1,n}$ is a Super beta combination graph.

Proof: Let $K_{1,n}$ be the star graph with $n+1$ and n edges.

Let $V(K_{1,n}) = \{u_i : 1 \leq i \leq n+1\}$. Let $E(K_{1,n}) = \{u_i u_{n+1} : 1 \leq i \leq n\}$.

Define a bijection $f : V(K_{1,n}) \to \{1,2,3,\ldots, n+1\}$ by $f(u_{n+1}) = 1$;

$f(u_i) = i$ *if* $2 \leq i \leq n$. And f induces that $f^s : E(K_{1,n}) \to N$, where N is a natural number, by $f^s(uv) = \dfrac{[f(u) + f(v) - 1]!}{[f(u) - 1]![f(v) - 1]!}$, for every edges uv of $K_{1,n}$ and are all distinct. That is f^s is injective. Hence star graph $K_{1,n}$ is a Super beta combination graph.

Example 2.2. A Super beta combination labeling of $K_{1,8}$ is shown in the Figure-2.1.

F**igure-2.2**

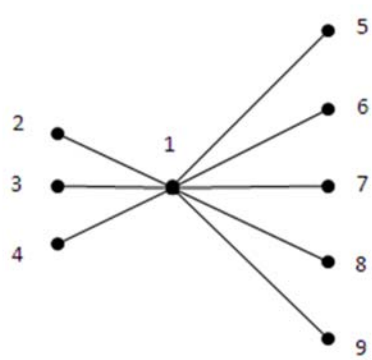

Figure-2.1

Theorem 2.3. The graph $P_m \cup P_n$ h.

Proof: Let P_m be a path graph with m vertices and $m-1$ edges.

Let $V(P_m)=\{u_i : 1\le i\le n\}$. Let $E(P_m)=\{u_iu_{i+1} : 1\le i\le m-1\}$.

Let P_n be a path graph with n vertices and $n-1$ edges.

Let $V(P_n)=\{v_i : 1\le i\le n\}$. Let $E(P_n)=\{v_iv_{i+1} : 1\le i\le n-1\}$.

Let $P_m\cup P_n$ be the union graph of paths P_m and P_n with $m+n$ vertices and $m+n-2$ edges.

Let $V(P_m\cup P_n)=\{u_i : 1\le i\le m;\ v_i : 1\le i\le n\}$.

Let $E(P_m\cup P_n)=\{u_iu_{i+1} : 1\le i\le m-1;\ v_iv_{i+1} : 1\le i\le n-1\}$.

Define a bijection $f:V(P_m\cup P_n)\to\{1,2,3,\ldots,m+n\}$ by $f(u_i)=i$ *if* $1\le i\le m$ and $f(v_i)=m+i$ *if* $1\le i\le n$. And f induces that $f^s:E(P_m\cup P_n)\to N$, where N is a natural number, by $f^s(uv)=\dfrac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$, for every edges uv of $P_m\cup P_n$ and are all distinct. That is f^s is injective. Hence the graph $P_m\cup P_n$ is a Super beta combination graph.

Example 2.4 . A Super beta combination labeling of path $P_5\cup P_6$ is shown in the Figure-2.2.

Figure-2.2

Theorem 2.5. Every double triangular snake $D(T_n)$ is a Super beta combination graph.

Proof: Let $D(T_n)$ be a double triangular snake with $3n-2$ vertices and $5n-5$ edges.

Let $V(D(T_n))=\{u_i : 1\le i\le n;\ v_i, w_i : 1\le i\le n-1\}$.

Let $E(D(T_n))=\{u_iu_{i+1}, u_iv_i, u_iw_i, u_{i+1}v_i, u_{i+1}w_i, : 1\le i\le n-1\}$.

Define a bijection $f:V(D(T_n))\to\{1,2,3,\ldots,3n-2\}$ by $f(u_1)=1$; $f(u_i)=3i$ *if* $2\le i\le n$; $f(v_i)=3i-1$ *if* $1\le i\le n-1$; $f(w_i)=3i+1$ *if* $1\le i\le n-1$.

And f induces that $f^s:E(D(T_n))\to N$, where N is a natural number, by $f^s(uv)=\dfrac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$, for every edges uv of $D(T_n)$ and are all distinct. That is f^s is injective. Hence the double triangular snake $D(T_n)$ is a Super beta combination graph.

Example 2.6. A Super beta combination labeling of double triangular snake $D(T_5)$ is shown in the Figure-2.3.

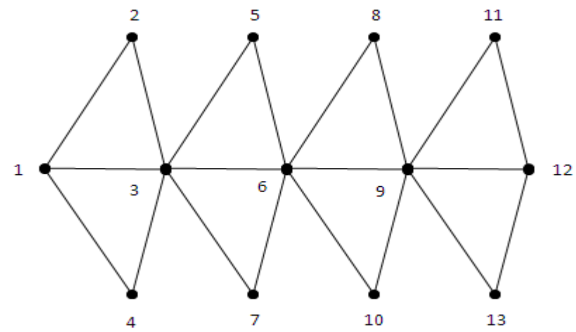

Figure-2.3

Theorem 2.7. Any graph $T_n \Theta K_1$ is a Super beta combination graph.

Proof: Let $T_n \Theta K_1$ be a graph with $4n-2$ vertices and $5n-4$ edges.

Let

$$V(T_n \Theta K_1) = \begin{cases} u_i \ :1 \le i \le n, \ v_i \ :1 \le i \le n-1 \\ w_i \ :1 \le i \le n, \ z_i \ :1 \le i \le n-1 \end{cases}.$$

Let $$E(T_n \Theta K_1) = \begin{cases} u_i u_{i+1} \ :1 \le i \le n-1; \ u_i v_i \ :1 \le i \le n-1 \\ u_{i+1} v_i \ :1 \le i \le n-1; \ u_i w_i \ :1 \le i \le n \\ v_i z_i \ :1 \le i \le n-1 \end{cases}$$

Define a bijection $f : V(T_n \Theta K_1) \to \{1,2,3,...,4n-2\}$ by $f(u_1) = 2$;

$f(u_i) = 3i+2$ if $2 \le i \le n$; $f(v_i) = 4i-1$ if $1 \le i \le n-1$; $f(w_1) = 1$

$f(w_i) = 4i+2$ if $2 \le i \le n$; $f(z_i) = 4i$ if $1 \le i \le n-1$.

And f induces that $f^s : E(T_n \Theta K_1) \to N$, where N is a natural number, by $f^s(uv) = \dfrac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$, for every edges uv of $T_n \Theta K_1$ and are all distinct.

That is f^s is injective. Hence any graph $T_n \Theta K_1$ is a Super beta combination graph.

Theorem 2.8. The graph $Q_n \Theta K_1$ admits a Super beta combination labeling.

Proof: Let $Q_n \Theta K_1$ be a graph with $6n-4$ vertices and $7n-6$ edges.

Let $$V(Q_n \Theta K_1) = \begin{cases} u_i \ :1 \le i \le n; \ \ v_i, w_i :1 \le i \le n-1 \\ x_i \ :1 \le i \le n; \ \ y_i, \ z_i \ :1 \le i \le n-1 \end{cases}.$$

Let $$E(Q_n \Theta K_1) = \begin{cases} u_i u_{i+1}, \ u_i v_i, \ u_{i+1} w_i, \ v_i w_i \ :1 \le i \le n-1 \\ u_i x_i \ :1 \le i \le n; \ v_i y_i \ :1 \le i \le n-1 \\ w_i z_i \ :1 \le i \le n-1 \end{cases}$$

Define a bijection $f : V(Q_n \Theta K_1) \to \{1,2,3,...,6n-4\}$ by $f(u_1) = 2$;

$f(u_i) = 6i+1$ if $2 \le i \le n$; $f(v_i) = 6i-3$ if $1 \le i \le n-1$;

$f(w_i) = 6i$ if $1 \le i \le n-1$; $f(x_1) = 1$; $f(x_i) = 6i+2$ if $1 \le i \le n$;

$f(y_i) = 6i-2$ if $1 \le i \le n-1$; $f(z_i) = 6i-1$ if $1 \le i \le n-1$.

And f induces that $f^s : E(Q_n \Theta K_1) \to N$, where N is a natural number, by

$f^{s}(uv)=\dfrac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$, for every edges uv of $Q_n\Theta K_1$ and are all distinct.

That is f^{s} is injective. Hence the graph $Q_n\Theta K_1$ admits a Super beta combination labeling.

Example 2.9. A Super beta combination labeling of graph $Q_4\Theta K_1$ is shown in the Figure-2.4.

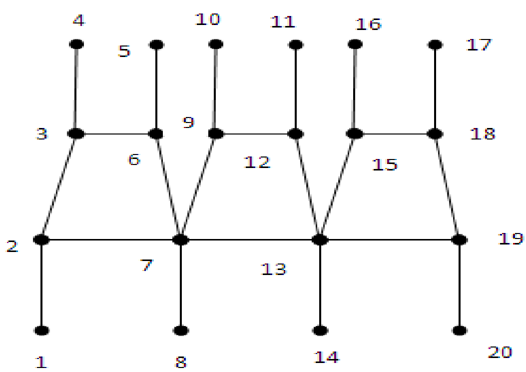

Figure-2.4

Theorem 2.10. Any graph $T_m \cup Q_n$ is a Super beta combination graph.

Proof: Let T_m be a triangular snake with $2m-1$ vertices and $3m-3$ edges.

Let $V(T_m)=\{u_i : 1\le i\le m \,;\, v_i : 1\le i\le m-1`\}$. Let $E(T_m)=\{u_iu_{i+1} , u_iv_i , u_{i+1}v_i : 1\le i\le m-1\}$.

Let Q_n be a quadrilateral snake with $3n-2$ vertices and $4n-4$ edges.

Let $V(Q_n)=\{w_i : 1\le i\le n \,;\, x_i , y_i : 1\le i\le n-1\}$.

Let $E(Q_n)=\{w_iw_{i+1} , w_ix_i , w_{i+1}y_i , x_iy_i : 1\le i\le n-1\}$.

Let $T_m \cup Q_n$ be a graph with $3m+3n-5$ vertices and $3m+4n-7$ edges.

Let $V(T_m\cup Q_n)=\begin{cases} u_i : 1\le i\le m \,;\, v_i : 1\le i\le m-1 \\ w_i : 1\le i\le n \,;\, x_i, y_i : 1\le i\le n-1 \end{cases}$.

Let $E(T_m\cup Q_n)=\begin{cases} u_iu_{i+1} , u_iv_i , u_{i+1}v_i : 1\le i\le m-1 \\ w_iw_{i+1} , w_ix_i , w_{i+1}y_i , x_iy_i : 1\le i\le n-1 \end{cases}$.

Define a bijection $f : V(T_m \cup Q_n) \rightarrow \{1,2,3,...,3m+3n-5\}$ by

$f(u_i)=2i-1$ if $1\le i\le m$; $f(v_i)=2i$ if $1\le i\le m-1$; $f(w_i)=2m-3+3i$ if $1\le i\le n$;
$f(x_i)=2m-2+3i$ if $1\le i\le n-1$; $f(y_i)=2m-1+3i$ if $1\le i\le n-1$.

And f induces that $f^{s} : E(T_m\cup Q_n)\rightarrow N$, where N is a natural number, by $f^{s}(uv)=\dfrac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$, for every edges uv of $T_m\cup Q_n$ and are all distinct.

That is f^{s} is injective. Hence any graph $T_m\cup Q_n$ is a Super beta combination graph.

Theorem 2.11. The graph $(P_m\Theta K_1)\cup(C_n\Theta K_1)$ admits a Super beta combination labeling.

Proof: Let $P_m\Theta K_1$ be a comb graph with $2m$ vertices and $2m-1$ edges.

Let $V(P_m\Theta K_1)=\{u_i\ :1\le i\le m\ ;\ v_i:1\le i\le m\}$.

Let $E(P_m\Theta K_1)=\{u_iu_{i+1}\ :1\le i\le m-1\ ;\ u_iv_i:1\le i\le m\}$. Let $C_n\Theta K_1$ be a crown with $2n$ vertices and $2n$ edges. Let $V(C_n\Theta K_1)=\{w_i\ :1\le i\le n\ ;\ z_i:1\le i\le n\}$.

Let $E(C_n\Theta K_1)=\{w_iw_{i+1}\ :1\le i\le n-1\ ;\ w_1w_n\ ;\ w_iz_i:1\le i\le n\}$

Let $P_m\Theta K_1\cup C_n\Theta K_1$ be the union graph with $2m+2n$ vertices and $2m+2n-1$ edges.

Let $V(P_m\Theta K_1\cup C_n\Theta K_1)=\begin{cases} u_i\ ,\ v_i:1\le i\le m \\ w_i\ ,\ z_i:1\le i\le n \end{cases}$.

Let $E(P_m\Theta K_1\cup C_n\Theta K_1)=\begin{cases} u_iu_{i+1}:1\le i\le m-1;\ \ u_iv_i:1\le i\le m \\ w_iw_{i+1}:1\le i\le n-1; w_1w_n\ ;\ w_i\ z_i:1\le i\le n \end{cases}$.

Define a bijection $f:V(P_m\Theta K_1\cup C_n\Theta K_1)\to\{1,2,3,...,2m+2n\}$ by $f(u_i)=2i-1\ \ if\ \ 1\le i\le m$; $f(v_i)=2i\ \ if\ \ 1\le i\le m$; $f(w_i)=2m+n+i\ \ if\ \ 1\le i\le n$; $f(z_i)=2m+i\ \ if\ \ 1\le i\le n$. And f induces that $f^s:E(P_m\Theta K_1\cup C_n\Theta K_1)\to N$, where N is a natural number, by $f^s(uv)=\dfrac{[f(u)+f(v)-1]!}{[f(u)-1]![f(v)-1]!}$, for every edges uv of $(P_m\Theta K_1)\cup(C_n\Theta K_1)$ and are all distinct. That is f^s is injective. Hence the graph $(P_m\Theta K_1)\cup(C_n\Theta K_1)$ admits a Super beta combination labeling.

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