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Influence of students reasoning patterns on achievement in Senior Secondary School Plane Geometry: Implications for Biology Education Practitioners

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Abstract

Reasoning patterns are an important part of education that are required for understanding and solving plane geometric problems. Mathematics teachers should use appropriate teaching methods to maintain a high level of reasoning pattern.This study determined the influence of students reasoning pattern on academic achievement in plane geometry. The researchers adopted ex-post facto research design. The study participants was made up of 368 mathematics students (180 males and 188 females) drawn using multi-stage sampling procedure. Instruments used for data collection were Plane Geometry Achievement Test (PGAT) and Plane Geometry Reasoning Pattern Classification Test (PGRPCT). The reliability of the instrument was determined using Kudder-Richardson formula (KR_{20}) method, and reliability estimate of 0.81 was obtained. The reliability of PGRPCT was ascertained using Pearson Product Moment Correlation Coefficient, the reliability estimate of 0.88 was obtained. The research questions were answered using mean and standard deviation while the null hypotheses were tested at $P< 0.05$ using analysis of variance (ANOVA). The result of the study revealed that mathematics students employed all the two reasoning patterns while solving geometric problems. There was a significant difference in the mean achievement scores of students in all the two levels of reasoning patterns. The teacher should use appropriate instructional design to ensure that students who give a correct verbal description of a geometric concept also has the correct concept image associated with that concept.

Keywords: Students reasoning pattern, visualization, analysis, academic achievement, mathematics, plane geometry

Introduction

Mathematics is widely regarded as the science of numbers, shapes, properties and relations of points and patterns. It is the science of patterns, such as counting patterns, reasoning patterns, communicating patterns, motion patterns, change patterns, symmetry patterns and regularity patterns, and position patterns (Petti, 2015). The author revealed that, mathematics involves a logical expression of relationships that exist among the measurable quantities of time and space in the universe, presented in compact and simple codes. Mathematics is a science that deals with measurement, the logic of shape, quantity, arrangement and patterns. Mathematics can be applied in all core sciences such as Biology, Physics, Chemistry; Social Sciences, such as Economic Psychology and Sociology; Engineering fields, such as civil, industrial and mechanical engineering; technological field, such as computers, rockets, accountancy, medicine, technology, weather forecasting, banking and data processing (Elaine, 2013).

Mathematics has been a pivot on which other subjects, especially science subjects, revolve. An individual has to be well equipped with the basic knowledge of mathematics in order to operate effectively and apply it in other school subjects like Chemistry, Physics, Biology, Economics, Geography and among others (Kurumeh & Dogo, 2015). The study of Mathematics help students develop skills like logical and rigorous argumentation, abstract thinking, formulating and solving problems, analyzing data, creating and analyzing mathematical models, communication, conceptual ability, interpretation, and research (Agwagah, 2013). Agwagah further stated that any nation's overall development and the creation of a healthy, happy, and prosperous society are impossible to achieve without the knowledge of mathematics. As a result, better achievement in mathematics could lead to better national and technological development. Despite this, students struggle with mathematics. Evidence from the Chief Examiner's reports on students' achievement in senior school certificate examinations (SSCE) in mathematics between 2015 and 2018 shows that students' achievement in mathematics is declining. Take for instance, in 2015, 1,605,248 sat for the examination and 620,910 students passed while 984,338 students failed (WAEC, 2015). In 2016, 1,552,758, students sat for the examination and 822,496 students passed while 730,262 students failed (WAEC, 2016). In 2017, 1567,016 students sat for the examination, 927,987 students passed while 630,029 students failed (WARC,2017). In 2018, 1,578,846 students sat for the examination and 789,107 students passed while 789,739 students failed (WAEC, 2018).

Students' achievement in Mathematics has remained poor for many years (Agwagah & Utibe, 2015; Wale, 2015; Ibrahim, 2012). This situation is due to students' lack of understanding of mathematical formulae or concepts, which is a result of the structure of knowledge and reasoning patterns they acquire during the mathematics teaching and learning processes. Students use these knowledge structures and reasoning patterns to provide solutions to mathematical problems. Unqualified mathematics teachers, according to Zalmon and Wonu (2017), are to blame for students' poor achievement in the West African Examination Council and National Examinations Council in successive years. Several factors influence students' achievement in mathematics. Such as: Poor teaching methods (Josiah & Etuk-Iren, 2014); teachers' inability to use appropriate instructional materials and lack of infrastructural facilities; students' poor reading habits (Hassan, 2014); lack of orientation on the relevance of mathematics to life in general and gender differentials (Bello, 2014) are some of the factors that can be attributed to poor teaching methods. Both students at various levels of learning, according to Ajani and Papoola (2013), complained bitterly about the difficulty of learning mathematics. Sheras (2014) observed that factors responsible for failure and consistent poor achievement have been identified, including: poor teaching approach and the non-use of adequate methods in teaching some concepts in mathematics, such as linear equations, quadratic equations, simultaneous equations, geometry.

Geometry is a branch of mathematics concerned with the study of figures, shapes, sizes, patterns, and locations. Geometry, according to Mukati (2016), is a branch of mathematics concerned with the properties, measurements, and relationships of points, lines, angles, surfaces, and solids. Geometry is the visual study of shapes, sizes, and how they fit together in space. Geometry has many sub-disciplines. Analytic geometry, differential geometry, topology, non-Euclidean geometry, and plane geometry are the five types of geometry. Geometry generally, is a very broad topic both in primary and secondary school levels, but the researchers could not explore all the levels. Therefore, the researchers dealt with plane geometry only with SS1 mathematics students. Plane geometry has been identified as a difficult topic by Alex and Mammen (2016); Rizki, et al. (2018) due to teachers' failure to use appropriate methods and patterns in teaching some concepts. Plane Geometry is the geometry of points, lines, curves, and other plane shapes, as well as their constructions. Plane geometry is two-dimensional geometry. Plane shapes are also two-dimensional shapes with straight lines or curves to describe them. Triangles, quadrilaterals, circles, and other polygons are examples of plane shapes. According to Howse and Howse, (2015), students could grasp the concept of geometry as well as plane geometry if they had good reasoning patterns. Wale, (2015) was in opinion with Jonah and Dogo, (2019) that students viewed geometry as difficult, theoretical, abstract, too formal, and complicated due to poor reasoning pattern. According to Wale, geometry as a branch of mathematics is in bad state of teaching and learning. Wale also emphasizes that the teaching of geometrical content, as practiced by today's mathematics teachers, has been found to be ineffective, resulting in poor student achievement.

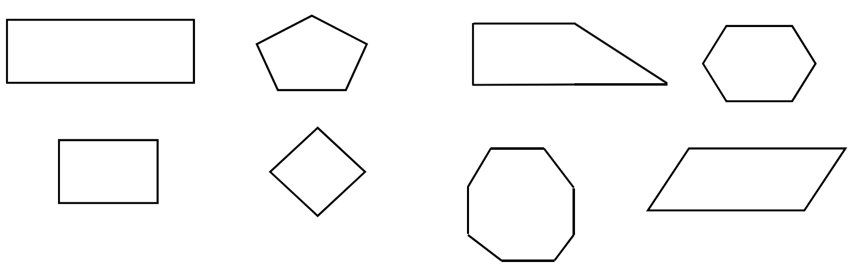
Many factors have been identified and attributed to the students' poor achievement in geometry. Such factors are: curriculum planners' inability to incorporate geometric reasoning patterns into the new curriculum design; examining bodies' lack of orientation and misconceptions about the relevance of reasoning patterns to life in general; teachers' unpreparedness; poor teaching environment; and unavailability of textbooks to mention just a few of the factors (Dangpe, 2015; Telima, 2014). However, available textbooks on geometry, according to Charles-Ogan and Nechelem (2015), emphasize abstract reasoning or cite experiences that are not common to the majority of students. These concepts/topics do not relate to the cultural experiences of African students, rather include those of the foreign countries. As a result, some students rely solely on what is done in class because they find it difficult to comprehend what is written in textbooks and library books. According to Olaleye (2015), students lack the necessary methods and patterns for studying geometry as an important aspect of mathematics, which has an impact on their achievement in geometry.

Teachers' incorrect presentation of geometric facts and solutions, without allowing students to provide their own concrete materials of many geometric shapes, interact geometrically with those shapes and develop their reasoning patterns, may be one of the persistent causes of students' poor achievement in geometry (Bankov, 2013). A good mathematics teacher should concentrate on geometric concepts and use problems that are challenging enough for students to develop their geometric reasoning pattern (Pournara, et al., 2013). The need to investigate appropriate methods that could improve students' geometry achievement has thus remained a critical issue for mathematics educators. The report of the West African Examination Council Chief Examiners (2019) revealed that students struggled with geometrical concepts in mathematics. Due to faulty reasoning patterns identified by Wahyudin as cited in Mikrayanti, (2016), students may try to use the correct problem-solving method but take the wrong path when looking for solutions. From all indications, students have difficulty in understanding certain geometric concepts due to lack of appropriate reasoning patterns. Even when students are given accurate instruction, they tend to structure it differently and distort it as a result of their poor reasoning patterns.

Plane geometry uses reasoning patterns, which acts as the main contributor towards achievement. Reasoning is the process of using evidence to determine conclusions. Reasoning, according to Bieda, et al. (2013), is the process by which a learner expresses and demonstrates connections between patterns of representations. A pattern is a set of instructions or a regular arrangement of shapes, lines, or both. In plane geometry, there are two types of patterns: repeated patterns and growth patterns. The term "repeated pattern" refers to a pattern with discernible units of repetition. Students are able to perceive, recognize, and distinguish something from something else. Each term in the pattern is dependent on the previous term and its position in the pattern, and each term in the pattern has discernible units known as terms. Students can be challenged to translate the visual, auditory, and movement of growing patterns once they have solidified their understanding of the rules. It is the evolving patterns that lead to the emergence of new ones. Teachers should motivate students by asking open-ended questions that encourage them to think beyond the patterns provided or what can be developed with additional materials. Karen (2012) went on to say that curriculum planners should examine the curriculum and methods of teaching geometry, particularly plane geometry, in various educational sectors and arrange them in a logical order based on reasoning patterns.

Reasoning patterns are an important part of education that are required for understanding and solving plane geometric problems. Mathematics teachers should use appropriate teaching methods to maintain a high level of reasoning pattern. Kanimozhi and Ganesan (2017) found that some students' reasoning patterns are very high, high, average, poor, and very poor. The researchers will identify students who have very high, high, low, very low, poor, and very poor levels of reasoning patterns in the Nigerian context in this study. When it comes to solving plane geometry problems, there are a variety of reasoning patterns to choose from. Hieles' (1986) theory of learning geometry identified five types of reasoning patterns in plane geometry, which have sparked a lot of interest around the world. The learner cannot successfully complete one level of reasoning without having completed the previous levels. According to Hieles, students go through five different types of reasoning patterns (Visualization, analysis, theoretical, deduction, and rigor are the five elements) but this study will concentrate only on two reasoning patterns (Visualization & Analysis).

Visualization is the process of using a chart or other image to represent or interpret an object, shape, situation, or set of instructions. This is the reasoning pattern in which a student presents geometrical ideas using language, notation, and geometrical structures. According to Hassan (2015), students name only a subset of shape visual characteristics, distinguish, compare, and operate on geometrical shapes such as triangles, angles, and parallel lines based on their appearances. It also entails the student's interpretation of images, maps, and sketches, as well as recognition from various perspectives. When asked why a figure is a square, a student might respond, "it looks like a square" or "it's like a cube of sugar." Students who use visualization to reason can recognize shapes based on their appearance but not specifically properties. Teacher uses inquiry, illustrations, questioning, demonstration to ensure mastery of different orientation of shapes before moving to the analysis pattern. The reasoning pattern in which a student uses language, notation and geometric structures to present geometrical ideas. It also involves the students' identification, characterization, interpretation and sorting shapes of plane geometric figures. For example, students identify, name, compare shapes and patterns of the following plane shapes:



Analysis is the breaking down concepts into their constituent parts or examining various objects to discover all of their properties. This involve sorting and classifying shapes in terms of properties. Students identify shapes given various groupings of properties and examine each element of a plane shapes or features of it in detail in order to understand it (Abu & Nimrawi, 2014). Teachers should ensure that students construct shapes and identify the properties, and use the properties to solve problems. Students can analyze the properties of figures, for Students identify shapes given various groupings of properties and examine each element of a plane shapes or features of it in detail in order to understand it. Teachers should ensure that students construct shapes and identify the properties, and use the properties to solve problems. Students can analyze the properties of figures, for example, rectangles have equal diagonals, it has four right angles and opposite sides are equal and a rhombus has all sides equal. Another example is that an equilateral triangle can have three equal sides, three equal angles and three axes of symmetry, in a circle where two radii are joined by a chord, an isosceles triangle is formed. It takes time to move from one level to the next. The researchers therefore considers it worthwhile to ascertain whether reasoning patterns can influence student's achievement in geometry, since the reasoning patterns are what the students actually use to provide answers to questions posed to them in geometry.

Theoretical Framework

This study is anchored on the theory of Geometric Reasoning propounded by Van Hiele and Van Hiele (1986). Van Hiele and Van Hiele-Gelgof were husband and wife researchers who studied school children's reasoning and concept development in geometry. They noticed that their students had difficulty learning geometry as a result of their many years of teaching experience. According to Van Hieles, when it comes to classroom observation and interactions, students go through several categories or levels of reasoning when it comes to geometric concepts. As a result, the Van Hieles developed the Van Hiele theory, which is a theory of categories of reasoning in geometry. Hieles was particularly interested in how students' progress from recognizing shapes to being able to construct a formal geometric proof through various levels or categories of geometric reasoning. Hieles believed that man's ability to do abstract symbolic reasoning distinguishes him from other animals created by God. The theory has two major components. The Van Hiele theory explains why many students struggle in geometry classes, especially when it comes to formal proofs. Second, the Van Hiele theory of geometry instruction, which provides a teaching model that teachers can use to improve students' geometric understanding. The literature uses two different numbering schemes to describe the Van Hiele patterns or levels of reasoning: level 0 through 4, and level 1 through 5.

Van Hiele identified five sequential and hierarchical levels of geometric reasoning: Level 1: Visualization, Level 2: analysis, Level 3: Theoretical, Level 4: Deduction and Level 5: Rigour. **Visualization** level: The student recognizes geometric shapes, identify, name, compare geometric shapes and also, produce a given shape. The student reasons by means of visual considerations without regard to the properties of its components.

Analysis level: The student at this level reasons and analyze geometric shapes in terms of its properties. The relationships between these properties and between different figures are not yet understood. **Theoretical**: The student at this stage, logically orders figures and understands interrelationships between different figures. Students use properties that they already know to formulate accurate definitions of geometric shapes. **Deduction**: The student understands the importance of deduction and the role of axioms, theorems and proofs. Students should be able to supply reasons for steps in a proof and as well construct their own proofs. **Rigor**: Students understand and reason formally about geometrical systems. The student understands the analyses of various theorems in different axiomatic systems and as well study various geometries in the absence of concrete models. According to Van Hiele, the levels or categories are sequential and hierarchical valid with higher levels of thinking which proceed under the influence of learning. Van Hiele used primary school children. The theory is related to the present study in the sense that the categories of geometric reasoning were also designed by the level of complicity, from simple to complex.

Evidence from literature and researches carried out in foreign countries also attributed the poor achievement in Mathematics to the reasoning patterns which students use to provide answers to questions on geometrical problems. Consequently, there is need to improve on the teaching and learning of geometry by exploring how to adapt reasoning patterns in the teaching of geometry. Therefore, the main focus of this study is to identify the various reasoning patterns employed by mathematics students while solving plane geometric problems and determine their influence on students' achievement. The purpose of the study was to investigate the influence of students levels of visualization and analysis reasoning patterns on their achievement in plane geometry.

Research Questions

The following research questions were addressed;

1. What are the mean achievement scores of students with different levels of visualization reasoning pattern in Plane Geometry Achievement Test (PGAT)?
2. What are the mean achievement scores of students with different levels of analysis reasoning pattern in PGAT?

Methodology

Design of the Study

The researchers adopted Expost-facto research design. Expost-facto research design seeks to establish cause-effect relationships but differs from experimental study in that the researcher usually has no control over the variables of interest and therefore cannot manipulate them (Nworgu, 2015). The design was adopted because the researchers did not manipulate or control any of the variables. It is a design where some effects are attributed to some cause without any attempt to manipulate the independent variables. Hence, expost-facto research design is ideal for this study since the hypotheses and variables can be analyzed without manipulation of the variables (Simon & Goes, 2013)

Participants

The population of the study comprised four thousand, five hundred and fifty-nine, (4559) senior secondary one (SS1) mathematics students. The SS1 students were used for this study because plane geometry is in the SS1 curriculum and is a foundation class. Introducing geometry reasoning at this stage develops students logical thinking in fundamental aspects of geometry topics they are going to study in SS2 and SS3. The sample of this study comprised three hundred and sixty-eight (368) SS1 mathematics students, consisting of 180 males and 188 females, from thirty-one (31) schools. Sampling was done using multi-stage sampling procedure.

Research Instruments

Two instruments developed by the researchers for data collection were used in this study. The instruments were Plane Geometry Reasoning Pattern Classification Test (PGRPCT) and Plane Geometry Achievement Test (PGAT). PGRPCT was developed based on each reasoning pattern. Each reasoning pattern has a plate which contains sets of figures followed with items. Plate A is visualization reasoning pattern, with 9 figures and five questions. Plate B is on analysis reasoning pattern, with 8 figures and five questions. PGRPCT was used solely in classifying the students into different levels of each reasoning pattern. In each reasoning pattern, the score of each student was converted to 100%. The students' scores in percentage was used in classifying students into five levels of reasoning patterns. The test scores from (81-100%), (61-80%), (41-60%), (21-40%), (0-20%) was used as a basis for categorizing the students into very high, high, average, poor and very poor reasoning patterns respectively (Kanimozhi & Ganesan, 2017).

Plane Geometry Achievement Test (PGAT) is a 30- item multiple choice questions with four response options that range from A-D from which the students select response that best answers the question. PGAT was used to determine students' achievement in plane geometry. The test was developed from the specified SS1 mathematics contents with respect to the test blue print for the contents constructed by the researchers. The PGAT cover the following areas; triangle, quadrilaterals, congruency, polygons and circle. The instrument was scored on the basis that each correct response was assigned one (1) mark while incorrect response attracts a score '0'.

Validity and Reliability

The instruments were exposed to both content and face validation by three specialists whose disciplines were very relevant to the study. The specialists were specifically requested to scrutinize the items of the instrument in relation to the research purposes, ambiguity of words, test language used in constructing the items, and the structure and clarity of items. These guided them in making useful criticisms and contributions to the instrument. The content validity of Plane Geometry Achievement Test (PGAT) was constructed with strict adherence to the test blue print so as to ensure that the test items reflect all details on the test blue print. The face validation of PGRPCT focused on the item arrangement and logical sequence. The specialists' comments and suggestions helped in modifying the items to suit the problem under investigation.

The instruments were trial-tested on a sample of 20 SS1 mathematics students who were not among the sample size. The field tested results were used to estimate the coefficient of internal consistency of the instruments using Test retest estimate of temporary stability for PGRPCT and Kudder-Richardson (K-R_{20}) for PGAT. Specifically, PGRPCT was administered on two occasions to twenty students. The scores of the students in the first and second administration were correlated using Pearson moment correlation and coefficient of 0.88 was obtained. The internal consistency of PGAT was found to be 0.81. This means that the instruments are reliable and capable of eliciting information for the purpose of the study.

Research Procedure

Before the administration of the PGRPCT, the students were notified that the exercise was not an academic test, but rather, the researchers was only interested in finding their own personal viewpoints in respect to the questions being asked. The copies of PGRPCT was first distributed to the sampled students, with the approval of the school principals and the assistance of regular mathematics teachers in each of the sampled schools. PGRPCT took the average of 1 hour 30 minutes. The PGRPCT was used to categorize the students into different levels of reasoning patterns. The test scores from (81-100%), (61-80%), (41-60%), (21-40%) and (0-20%) was used as a basis for categorizing the students into very high, high, average, low, and very low levels of reasoning patterns respectively. The shapes of those four PGRPCT (Plate A - D) were presented to the students, one after the other. Students responses to each of the questions in PGRPCT were used to categorized them into different levels of the reasoning pattern. This was followed by Plane Geometry Achievement Test (PGAT). Before administration of the test, the researchers reminded each student sampled for the test that the test is just a follow-up exercise to the PGRPCT they earlier responded to. The researcher pleaded for honest responses and emphasized that there is no right or wrong answers. On the average, the students were allowed to respond to the instruments within 50 minutes.

Method of Data Analysis

Data collected were analyzed using SPSS version 23. Research questions were answered using mean and standard deviation. Mean and standard deviation were used because mean is the most reliable measuring tendency. Also, standard deviation is the most reliable estimate of variability. Data collected from PGRPCT were used for classification of students into various levels of reasoning patterns, while data collected from PGAT, were used to answer all the research questions. Hypotheses were tested using analysis of variance (ANOVA) at 0.05 level of significance. Analysis of variance was used because it is a statistical tool used to determine if at least one group mean is different from the others.

Results

Result in Table 1 shows the mean and standard deviations of students' achievement scores with various levels of visualization reasoning pattern in plane geometry. The result shows that students with extremely low visualization reasoning pattern in plane geometry had a mean achievement scores of $\bar{x}$ = 12.75, SD = 2.50, very low, $\bar{x}$ = 14.69, SD =1.60, low, $\bar{x}$ = 15.67, SD = 3.33, average, $\bar{x}$ = 17.36, SD = 3.15, high, $\bar{x}$ = 18.96, SD = 3.31 and very high, $\bar{x}$ = 20.09, SD = 3.74. The mean scores show that students with very high visualization reasoning pattern have the highest achievement mean score, followed by those with high, average, low, very low and extremely low visualization reasoning pattern in plane geometry. The higher the SD, the more the scattering of the individual scores from the mean. If the mean is high relative to the other group means, and the SD is equally high, then only few individual scores are high. But if the SD is small, it implies that most of the individual scores are around the mean, that is, the sores are nearly equal to the mean. Thus, the SD for each of the levels of visualization reasoning pattern indicates a low degree of variation in the scores of students with various level of visualization reasoning pattern.

Table 1. Mean achievement scores of students with different levels of visualization reasoning pattern in PGAT

Levels	N	$\bar{X}$	SD.	Std. Error
Extremely low	4	12.7500	2.50000	1.25000
Very Low	13	14.6923	1.60128	.44412
Low	6	15.6667	3.32666	1.35810
Average	36	17.3611	3.15461	.52577
High	121	18.9587	3.31008	.30092
Very High	131	20.0916	3.73847	.32663
Total	311	18.9293	3.72457	.21120

Key: $\bar{x}$ = Mean, SD = Standard Deviation,

The result in Table 2 shows the ANOVA comparison for achievement mean ($\bar{x}$) scores of students with different levels of visualization reasoning pattern in plane geometry. From the result, an F-value of 12.177 with an associated probability value of 0.000 was obtained. The result indicates that the associated probability value of 0.000 is less than the 0.05 level of significance. Thus, the null hypothesis which states that there is no significant difference in the mean achievement scores of students with different levels of visualization reasoning pattern in plane geometry is rejected. The inference drawn is that, there is significant difference in the mean achievement scores of students with different levels of visualization reasoning pattern in plane geometry.

Table 2. ANOVA comparison of the mean achievement scores of students with different levels of visualization reasoning pattern in PGAT

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	715.591	5	143.118	12.177	.000
Within Groups	3584.852	305	11.754		
Total	4300.444	310			

Key: df = degree of freedom, F = ANOVA test statistic, Sig. = Significant level/Exact probability value

Result in Table 3 shows the mean responses and standard deviations of students' achievement scores with various levels of analysis reasoning pattern in plane geometry. The result shows the sample size, mean score and standard deviations (n = 8, $\bar{x}$ = 21.88, SD = 2.59; n = 61, $\bar{x}$ = 19.74, SD = 3.62; n = 117, $\bar{x}$ = 19.56, SD = 3.58; n = 62, $\bar{x}$ = 18.34, SD = 3.68; n = 16, $\bar{x}$ = 17.75, SD = 3.34; n = 47, $\bar{x}$ = 16.98, SD = 3.63) for very high, high, average, low, very low and extremely low levels of analysis reasoning pattern of students in plane geometry respectively. The mean scores show that students with very high level of analysis reasoning pattern have the highest mean score, followed by those with high, average, low, very low and extremely low analysis reasoning pattern in plane geometry. The standard deviation for each of the levels of analysis reasoning pattern indicates a low variation in the scores of the students.

Table 3 Mean Achievement scores of students with different levels of analysis reasoning pattern in plane geometry

Levels	N	$\bar{X}$	SD	Std. Error
Extremely low	47	16.9787	3.63252	.52986
Very Low	16	17.7500	3.33667	.83417
Low	62	18.3387	3.67956	.46730
Average	117	19.5641	3.58008	.33098
High	61	19.7377	3.62354	.46395
Very High	8	21.8750	2.58775	.91491
Total	311	18.9293	3.72457	.21120

Key: $\bar{x}$ = Mean, SD = Standard Deviation,

Result in table 4 shows that an F-value of 5.898 with associated exact probability value of 0.00 was obtained for the difference in the mean achievement scores of students with different levels of analysis reasoning pattern in plane geometry. The result shows that the exact probability value of 0.00 is less than the 0.05 level of significance. Thus, the null hypothesis is rejected. The conclusion drawn is that there is significant difference in the mean achievement scores of students with different levels of analysis reasoning pattern in plane geometry.

Table 4. ANOVA result for the difference in the mean achievement scores of students with different levels of analysis reasoning pattern in plane geometry

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	379.130	5	75.826	5.898	.000
Within Groups	3921.313	305	12.857		
Total	4300.444	310			

Key: df = degree of freedom, F = ANOVA test statistic, Sig. = Significant level/Exact probability value

Discussion

The researchers discovered that students with very high visualization reasoning pattern have the highest mean achievement score, followed by those with high, average, low, very low and extremely low visualization reasoning pattern in plane geometry. The findings also revealed that the standard deviation for each of the levels of visualization reasoning pattern indicates a low degree of variation in the scores of students with various levels of visualization reasoning pattern. This indicates improvement on student' ability to reason scientifically as a result of instruction. There was significant difference in the mean achievement scores for the different levels of visualization reasoning pattern in plane geometry. This result disagrees with the results of earlier studies carried out by Kanimozhi and Ganesean (2017), Wahyhini and Hadi (2019), who found in their separate studies, that there is no significant difference in the reasoning levels of students while solving geometry, but agrees with the findings of Hassan (2015) and Bhat (2016) who found that the difference in the mean achievement scores of students was significant. It may be concluded that 131 out of 311 samples used for the study are very high in visualization reasoning pattern. One hundred and twenty- one (121) students are good in visualization reasoning pattern. Thirty-six students are average in visualization reasoning pattern. Six (6) students are low in visualization reasoning pattern. Four (4) students are extremely low in visualization reasoning pattern. From the researcher's view, the majority of students had mastery of geometric shapes which is a prerequisite knowledge for other patterns of reasoning.

Students with very high level of analysis reasoning pattern have the highest mean scores, followed by those with high, average, low, very low and extremely low analysis reasoning pattern in plane geometry. The standard deviation for each of the levels of analysis reasoning pattern indicates a low variation in the scores of the students. Result from the test of hypothesis two showed that there is significant difference in the mean achievement scores of students with different levels of analysis reasoning pattern in plane geometry. The findings support the similar works on levels of geometric thinking according to Van Hiele's done by Taha (2015) but do not agree with the findings of Ibrahim (2014), Abu and Nimrawi (2014) who indicated that students do not exceed the first level. It can be concluded that out of 311 samples for the study, only 117 students score the average mark. The finding of this study therefore showed that, the number of students that scores very high and high is dropping, compared with the Table 1 above. These results can be attributed to the teacher's focus on the visualization or conceptual knowledge in which students can only name and identify shapes or figures. From the data analyzed only eight students out of 311 students used for the study, can correctly identify and explain the properties of plane geometric figures. The number of students that scored 61% and above were only 69 students out of 311 students used for the study. Most of the students were still at the average level. The researchers observed from the study that majority of the students find it difficult to correctly state the properties of plane geometric figures.

Implications for Biology Education Practitioners

The implication of this findings to biology education teachers is that they will have to teach the terminology associated with a given content area in biology, the biology teacher should use appropriate instructional design and instruction to ensure that students who give verbal description of a biology concept also has the correct image associated with that concept. Biology education practitioners are by the findings of this study expected to carry out more studies in area of reasoning patterns to facilitates full integration of these modes of instructions while teaching biology students. Biology students need preliminary explorations of the properties of biology before they can attempt to write a definition of it.

Conclusions

There is significant different in students achievement with different levels of visualization reasoning pattern in plane geometry. However, the significant different in the students reasoning pattern is situated between very high, high, average, low, very and extremely low level of visualization reasoning patterns in plane geometry. There is a significant difference in the achievement mean scores of students with different levels of analysis reasoning pattern in plane geometry. Hence, the result of the students shows that there is significant mean difference between very high, low, very low and extremely low but there is no significant difference among very high, and average as well as low, very low and extremely low analysis reasoning patterns in plane geometry. There is a significant difference in response of students with extremely low level, low, average, high and very high reasoning pattern but no significant difference exists between low and very low level. The content scope chosen for this study was restricted to topics on plane geometry. The findings may or may not be the same if similar study is conducted with topics chosen from other concepts in mathematics.

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