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TO FIND THE DOMATIC NUMBER OF AN INTERVAL GRAPH 'G' RELATED TO AN INTERVAL FAMILY 'I'

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Abstract: The domatic number is a fundamental graph parameter that measures the maximum number of pair wise disjoint dominating sets into which the vertex set of a graph can be partitioned. Interval graphs, arising from families of intervals on the real line. In this paper, we investigate the domatic number of interval graphs associated with interval families and establish new theoretical results based on their structural properties. We derive sufficient conditions and bounds for the domatic number by exploiting the intersection relationships among intervals and the corresponding graph structure.

Keywords: Interval graph, Interval family, dominating set, dominating number, Domatic partition, Domatic number.

Introduction: The history of graph theory may be specifically traced to 1735, when the Swiss mathematician Leonhard Euler solved the Konigsberg bridge problem. Graph theory in mathematics is the study of graphs. Graphs are one of the prime objects of study in discrete mathematics. In mathematics graph theory is the study of graphs, which are mathematical structures, used to model pair wise relations between objects. A graph structure is very suitable for representing relationships between objects in the abstract, and a large number of combinatorial problems can be modeled as problems on the graph structure. A partition of a graph is the division of the vertex set or edge set of a graph into disjoint subsets such that every vertex or node and edges or links belongs to exactly one subset and also a vertex partition of a graph G is a collection of non-empty pair wise disjoint subsets. Let us consider the vertex set

$V = \{v_1, v_2, ..., v_n\}$ and $v_1 \cup v_2 \cup ..... \cup v_n = V(G)$, $V_i \cap V_j = \emptyset \ \forall \ i \neq j$, where

$i = 1, 2, ...$ And $j = 1, 2, ...$. Let us consider the finite subset of V, i.e.,

$V_1 = \{v_1, v_2, v_3\}$ and $V_2 = \{v_4, v_5, v_6\}$ these are pair wise disjoint and together contain all vertices. In graph theory the partitions are fundamental for studying graph structure, connectivity and many optimization problems.

Preliminaries: Let $I = \{I_1, I_2, ..., I_n\}$ be an interval family, where each I_i is an interval on the real line and $I_i = [a_i, b_i]$, for $i = 1, 2, \ldots,$ n. Here a_i is called left end point and b_i is called the right end point. Without loss of generality, we assume that, all end points of the intervals are

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distinct numbers between 1and n. The intervals are named in the increasing order of their right end points. Let I = {1, 2, …., n} be the interval family with right end point labeling corresponding to the interval graph G.
The graph G is called an interval graph if there is one - to– one correspondence between the vertex set V and an interval family I. Two vertices of G are joined by an edge if and only if their corresponding intervals in I are intersect. That is if i = [a_i, b_i] and j = [a_j, b_j], then i and j will intersect, if $a_i < b_j$ or $a_j < b_i$, in this connection each interval is nothing but one vertex.
A subset D of V is said to be a dominating set of G if every vertex in V - D is adjacent to a vertex in D. A dominating set with minimum cardinality is said to be a minimum dominating set.
The cardinality of the dominating set of a graph G is called domination number of G. It is denoted by $\gamma(G)$.
A collection of partition {d_1, d_2, d_3, ….., d_k} of G is called domatic partition (DP) of G if 1) {d_1, d_2, d_3, ….., d_k} are pair wise disjoint. i.e., $d_1 \cap d_2 \cap d_3 \cap \ldots \cap d_k = \emptyset$
2) $\bigcup_{i=1}^{k} d_i = V(G)$ i.e., $d_1 \cup d_2 \cup d_3 \cup \ldots \cup d_k = V(G)$
3) Each di is a dominating set by definition the maximum possible number dominating sets in a domatic partition is domatic number DN(G).

Main theorems:

Theorem 1: Suppose a finite consecutive interval family (without multiple intervals) I = {I_2, I_3,……., I_n} and G be an interval graph corresponding to I and the vertices must be greater than or equal to 2 . i.e.,$\forall n \geq 2$. Then the domatic number DN(G) = 2.

Proof: Let I = {I_2, I_3, ……., I_n} be a finite consecutive interval family without multiple intervals, and G be an interval graph corresponding to I.

We know that a collection of partition of G is a set of {d_1, d_2, d_3, ….., d_k} is called domatic partition DP(G) if

1) {d_1, d_2, d_3, ….., d_k} are pair wise disjoint.i.e.,$d_1 \cap d_2 \cap d_3 \cap \ldots \cap d_k = \emptyset$,
Where d_1, d_2, d_3, ….., d_k are called dominating sets of G.
2) $\bigcup_{i=1}^{k} d_i = V(G)$ i.e., $d_1 \cup d_2 \cup d_3 \cup \ldots \cup d_k = V(G)$
3) Each d_i is a dominating set by definition the maximum possible number of dominating sets in a domatic partition is domatic number DN(G).

Let us consider I = {I_2, I_3, ……., I_n} be a consecutive interval family and where $I_1 \notin I$ and G be a graph of I.

Suppose the vertex set V(G) = {v_1, v_2, v_3, v_4, v_5, v_6}. Then the dominating sets are d_1 = {v_1, v_3, v_5} and d_2 = {v_2, v_4, v_6}. It is clear that d_1 and d_2 are pair wise disjoint sets of G i.e., $d_1 \cap d_2 = \emptyset$ and $d_1 \cup d_2 = V(G)$ and every vertex in d_1 is adjacent with the vertices of d_2. And d_1, d_2 itself is a dominating sets. Therefore {d_1, d_2} is a domatic partition.
Hence all the three conditions are satisfied. So that the domatic number of the graph is 2.
Therefore, for every finite consecutive interval family with out multiple intervals have the domatic number DN (G) = 2.

Hence the theorem is proved.

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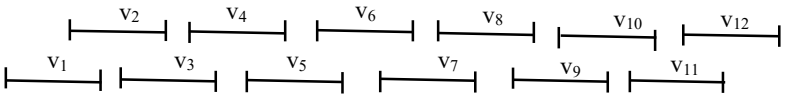
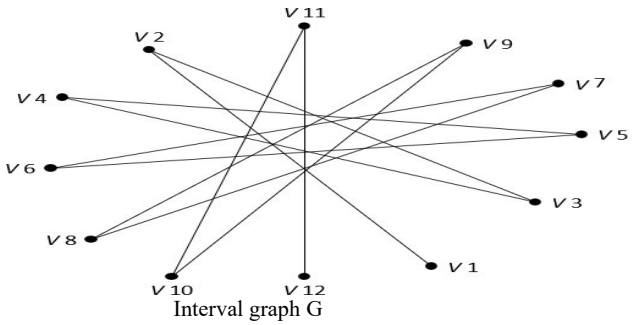
ILLUSTRATION:

n = 12:



Interval family I



Interval graph G

The dominating sets are $d_1 = \{v_2, v_4, v_5, v_7, v_8, v_{11}\}$ and $d_2 = \{v_1, v_3, v_6, v_9, v_{10}, v_{12}\}$

$d_1 \cap d_2 = \{v_2, v_4, v_5, v_7, v_8, v_{11}\} \cap \{v_1, v_3, v_6, v_9, v_{10}, v_{12}\}$

$d_1 \cap d_2 = \emptyset$ and

$d_1 \cup d_2 = \{v_2, v_4, v_5, v_7, v_8, v_{11}\} \cup \{v_1, v_3, v_6, v_9, v_{10}, v_{12}\}$

$d_1 \cup d_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$

$d_1 \cup d_2 = V(G)$

Therefore the domatic partition $DP(G) = \{d_1, d_2\}$

$\Rightarrow DN(G) = 2$.

Theorem 2:

If a finite even and odd consecutive interval family (without multiple intervals) $I = \{I_2, I_3, \ldots\ldots, I_n\}$ and G be an interval graph corresponding to I, and the vertices must be greater than or equal to 2 i.e., $\forall n \geq 2$ then the domatic number

$$DN(G) = 2 = \begin{cases} n-2k \text{ if } n = 2m, k = \frac{(n-2)}{2}, \forall k \in W, \forall m \in N \\ n-(2k+1) \text{ if } n = 2m+1, k = \frac{(n-3)}{2}, \forall k \in W, \forall m \in N \end{cases}$$

Proof:

Let $I = \{I_2, I_3, \ldots\ldots, I_n\}$ be a finite even and odd consecutive interval family (without multiple intervals) and G be a interval graph corresponding to I.

Let us suppose $I_E, I_O \in I$ where

$I_E = \{I_2, I_4, \ldots\ldots, I_{2m}\}$ = even and $I_O = \{I_3, I_5, \ldots\ldots, I_{2m+1}\}$ = odd and $\forall m \in N$

Since $I_E \cup I_O = I = V(G)$ and $I_E \cap I_O = \emptyset$. The graph must be connected at I_E, I_O.

We know that a collection of partition of G is set of $\{d_1, d_2, \ldots\ldots, d_k\}$ is called domatic partition DP(G) if

1) $\{d_1, d_2, d_3, \ldots\ldots, d_k\}$ are pair wise disjoint. i.e., $d_1 \cap d_2 \cap d_3 \cap \ldots\ldots \cap d_k = \emptyset$

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2) $\cup_{i=1}^{k} d_i = V(G)$i.e. , $d_1 \cup d_2 \cup d_3 \cup \ldots \cup d_k = V(G)$

3) Each d_i is a dominating set

By definition the maximum possible number of dominating sets in a domatic partition is domatic number DN(G).

For every finite consecutive interval family without multiple intervals has the domatic number DN(G) = 2.

In this , it will arise two cases

Case1:

Let us consider the finite even consecutive interval family $I_E = \{I_2, I_4, \ldots\ldots, I_{2m}\}$, $\forall m \in N$ without multiple intervals.

The domatic number DN(G) = n-2k where n = 2m, $k = \frac{n-2}{2}$, $\forall k \in W$, $\forall m \in N$

Let us consider the vertices at $I_E = \{v_1, v_2, \ldots\ldots, v_{2m}\}$ and $I_O = \{u_1, u_2, \ldots\ldots, u_{2m+1}\}$

Let n = 2 then the dominating sets are $d_1 = \{v_1\}$ and $d_2 = \{v_2\}$

Where $d_1 \cap d_2 = \{v_1\} \cap \{v_2\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_1\} \cup \{v_2\}$

$d_1 \cup d_2 = \{v_1, v_2\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore the domatic partition DP(G) = $\{d_1, d_2\}$.

Therefore domatic number DN(G) = n-2k

= 2-2(0)

= 2

Let n = 4 then the dominating sets are $d_1 = \{v_1, v_3\}$ and $d_2 = \{v_2, v_4\}$

Where $d_1 \cap d_2 = \{v_1, v_3\} \cap \{v_2, v_4\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_1, v_3\} \cup \{v_2, v_4\}$

$d_1 \cup d_2 = \{v_1, v_2, v_3, v_4\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore $\{d_1, d_2\}$ is a domatic partition of G.

Therefore domatic number DN(G) = n-2k

= 4-2(1)

= 2

Similarly n = 2m then the domatic number DN(G) = 2m-2k = 2

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$DN(G) = 2$

Case 2:

Let us consider the finite odd consecutive interval family $I_O = \{I_3, I_5, \ldots\ldots, I_{2m+1}\}$, $\forall m \in N$ without multiple intervals.

The domatic number $DN(G) = n - (2k+1)$ where $n = 2m+1$, $k = \frac{n-3}{2}$, $\forall k \in W$, $\forall m \in N$

Let us consider the vertices at $I_O = \{u_1, u_2, \ldots\ldots, u_{2m+1}\}$

Let $n = 3$ then the dominating sets are $d_1 = \{u_1\}$ and $d_2 = \{u_2, u_3\}$

Where $d_1 \cap d_2 = \{u_1\} \cap \{u_2, u_3\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{u_1\} \cup \{u_2, u_3\}$

$d_1 \cup d_2 = \{u_1, u_2, u_3\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself they are dominating sets. Therefore the domatic partition

$DP(G) = \{d_1, d_2\}$.

Therefore domatic number $DN(G) = n - (2k+1)$

$= 3-(0+1)$

$= 2$

Let $n = 5$ then the dominating sets are $d_1 = \{u_2, u_4\}$ and $d_2 = \{u_1, u_3, u_5\}$

Where $d_1 \cap d_2 = \{u_2, u_4\} \cap \{u_1, u_3, u_5\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{u_2, u_4\} \cup \{u_1, u_3, u_5\}$

$d_1 \cup d_2 = \{u_1, u_2, u_3, u_4, u_5\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets.

Therefore the domatic partition

$DP(G) = \{d_1, d_2\}$.

Therefore domatic number $DN(G) = n-(2k+1)$

$= 5-(2(1)+1)$

$= 5-3$

$= 2$

Similarly $n = 2m+1$ then the domatic number

$DN(G) = (2m+1) - (2k+1) = 2$

$DN(G) = 2$

Hence the theorem proof

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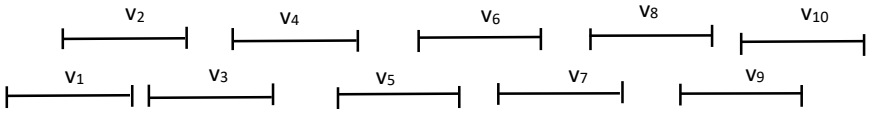
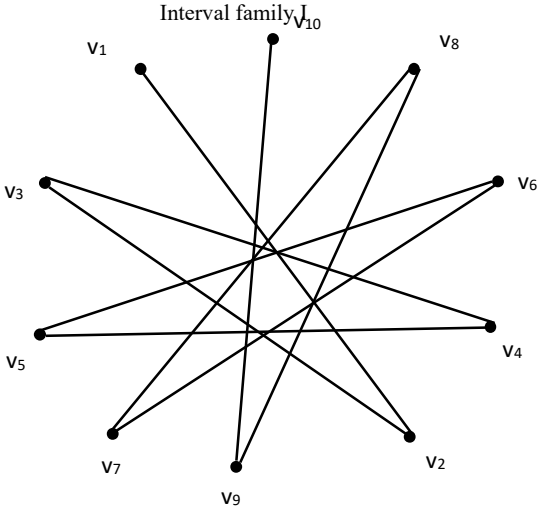
Illustration for even consecutive vertices:

n = 10:



Interval family I



Interval Graph G

The dominating sets are $d_1 = \{v_2, v_5, v_6, v_8, v_{10}\}$

$d_2 = \{v_1, v_3, v_4, v_7, v_9\}$

$d_1 \cap d_2 = \{v_2, v_5, v_6, v_8, v_{10}\} \cap \{v_1, v_3, v_4, v_7, v_9\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_2, v_5, v_6, v_8, v_{10}\} \cup \{v_1, v_3, v_4, v_7, v_9\}$

$= \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$

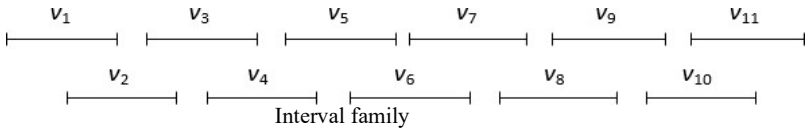
$= V(G)$

Therefore, $DP(G) = \{d_1, d_2\}$.

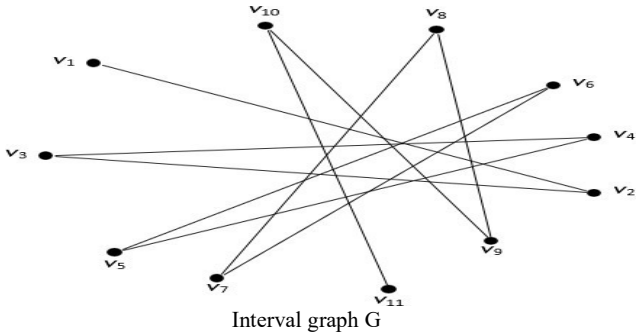
$\Rightarrow DN(G) = 2$

Illustration for odd consecutive vertices:

n = 11:



Interval family



Interval graph G

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The dominating sets are $d_1 = \{v_2, v_5, v_7, v_8, v_{10}\}$

$$d_2 = \{v_1, v_3, v_4, v_6, v_9, v_{11}\}$$

$d_1 \cup d_2 = \{v_2, v_5, v_7, v_8, v_{10}\} \cap \{v_1, v_3, v_4, v_6, v_9, v_{11}\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_2, v_5, v_7, v_8, v_{10}\} \cup \{v_1, v_3, v_4, v_6, v_9, v_{11}\}$

$= \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}\}$

$= V(G)$

Therefore $DP(G) = \{d_1, d_2\}$

$\Longrightarrow DN(G) = 2$

Theorem 3:

Let $I = \{I_2, I_3, \ldots, I_n\}$ be a finite interval family with even and odd contains and contained vertices, and G be an interval graph corresponding to I. The number of vertices must be greater than or equal to 2.Then the domatic number DN(G) = 2 except n = 3

Proof:

Suppose $I = \{I_2, I_3, \ldots, I_n\}$ be a finite interval family with even and odd contains and contained(no multiple intervals) vertices, and G be an interval graph corresponding to I.

We know that a collection of partition of G is set of $\{d_1, d_2, \ldots, d_k\}$ is called domatic partition of G if

1) $\{d_1, d_2, d_3, \ldots, d_k\}$ are pair wise disjoint, i.e., $d_1 \cap d_2 \cap d_3 \cap \ldots \cap d_k = \emptyset$
2) $\cup_{i=1}^{k} d_i = V(G)$ i.e., $d_1 \cup d_2 \cup d_3 \cup \ldots \cup d_k = V(G)$
3) Each d_i is a dominating set

By definition the maximum possible number of dominating sets in a domatic partition is domatic number DN(G).

In this it will arise two cases.

Case 1:

Let us consider the finite even contained and contains interval family

$I_E = \{I_2, I_4, \ldots\ldots, I_{2m}\}$, $\forall m \in N$ without multiple intervals.

The domatic number $DN(G) = n - 2k$ where $n = 2m$, $k = \frac{n-2}{2}$, $\forall k \in W$, $\forall m \in N$

Let us consider the vertices at $I_E = \{v_1, v_2, \ldots\ldots, v_{2m}\}$ and $I_O = \{u_1, u_2, \ldots\ldots, u_{2m+1}\}$

Let n = 2 then the dominating sets are $d_1 = \{v_1\}$ and $d_2 = \{v_2\}$

Where $d_1 \cap d_2 = \{v_1\} \cap \{v_2\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_1\} \cup \{v_2\}$

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$d_1 \cup d_2 = \{v_1, v_2\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore $\{d_1, d_2\}$ is a domatic partition(DP).

Therefore domatic number DN(G) = n-2k

$= 2\text{-}2(0) = 2$

Let n = 4 then the dominating sets are $d_1 = \{v_1, v_3\}$ and $d_2 = \{v_2, v_4\}$

Where $d_1 \cap d_2 = \{v_1, v_3\} \cap \{v_2, v_4\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_1, v_3\} \cup \{v_2, v_4\}$

$d_1 \cup d_2 = \{v_1, v_2, v_3, v_4\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore $\{d_1, d_2\}$ is a domatic partition.

Therefore domatic number DN(G) = n-2k

$= 4\text{-}2(1)$

$= 2$

Similarly n = 2m then the domatic number DN(G) = 2m-2k = 2

DN(G) = 2

Case2:

Let us consider the finite odd contained and contains interval family without multiple intervals.

$I_O = \{I_5, I_7, \ldots\ldots, I_{2m+1}\}$, $\forall m \in N$ without multiple intervals.

The domatic number DN(G) = n-(2k+1) where n = 2m+1, $k = \frac{n-3}{2}$, $\forall k \in W$, $\forall m \in N$

Let us consider the vertices at $I_O = \{u_1, u_2, \ldots\ldots, u_{2m+1}\}$

Let n = 5 then the dominating sets are $d_1 = \{u_2, u_4\}$ and $d_2 = \{u_1, u_3, u_5\}$

Where $d_1 \cap d_2 = \{u_2, u_4\} \cap \{u_1, u_3, u_5\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{u_2, u_4\} \cup \{u_1, u_3, u_5\}$

$d_1 \cup d_2 = \{u_1, u_2, u_3, u_4, u_5\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore $\{d_1, d_2\}$ is a domatic partition.

Therefore domatic number DN(G) = n - (2k+1)

$= 5 - (2(1)+1)$

$= 5\text{-}3$

$= 2$

Let n = 7 then the dominating sets are $d_1 = \{u_1, u_3, u_4, u_6\}$ and $d_2 = \{u_2, u_5, u_7\}$

Where $d_1 \cap d_2 = \{u_1, u_3, u_4, u_6\} \cap \{u_2, u_5, u_7\}$

$d_1 \cap d_2 = \emptyset$

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And

$d_1 \cup d_2 = \{u_1, u_3, u_4, u_6\} \cup \{u_2, u_5, u_7\}$

$d_1 \cup d_2 = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore $\{d_1, d_2\}$ is a domatic partition.

Therefore domatic number $DN(G) = n - (2k+1)$

$= 7 - (2(2)+1)$

$= 7-5 = 2$

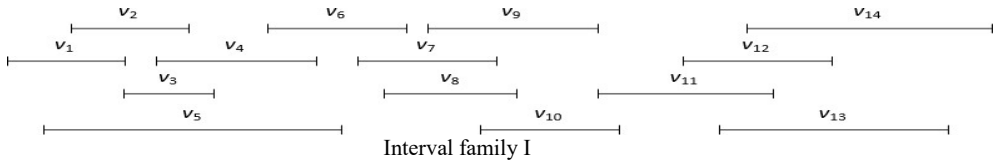
Similarly $n = 2m+1$ then the domatic number $DN(G) = (2m+1) - (2k+1) = 2$

$DN(G) = 2$

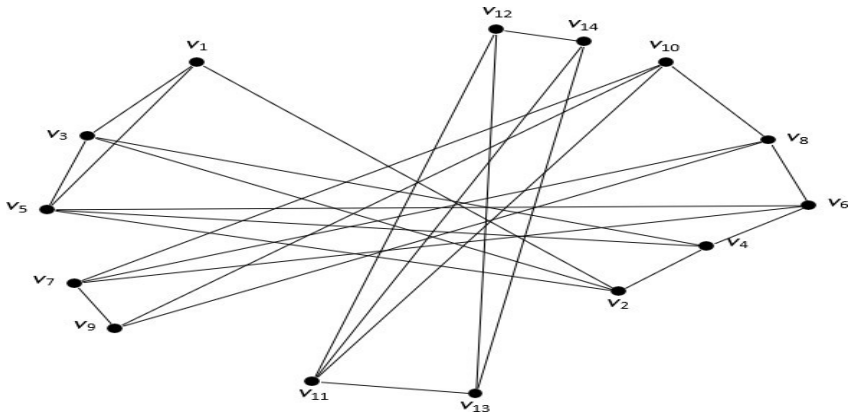
Hence the theorem proof

Illustration for even vertices:

n = 14:



Interval family I



Interval Graph G

The dominating sets are $d_1 = \{v_2, v_4, v_7, v_8, v_{10}, v_{12}, v_{14}\}$

$d_2 = \{v_1, v_3, v_5, v_6, v_9, v_{11}, v_{13}\}$

$d_1 \cap d_2 = \{v_2, v_4, v_7, v_8, v_{10}, v_{12}, v_{14}\} \cap \{v_1, v_3, v_5, v_6, v_9, v_{11}, v_{13}\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_2, v_4, v_7, v_8, v_{10}, v_{12}, v_{14}\} \cup \{v_1, v_3, v_5, v_6, v_9, v_{11}, v_{13}\}$

$= \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}\}$

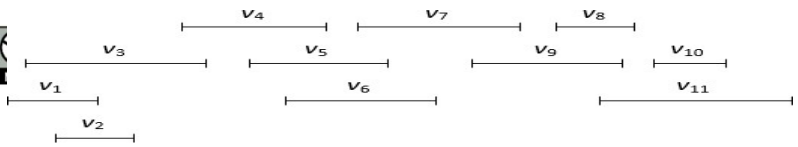
$= V(G)$

Therefore $\{d_1, d_2\}$ is a domatic partition

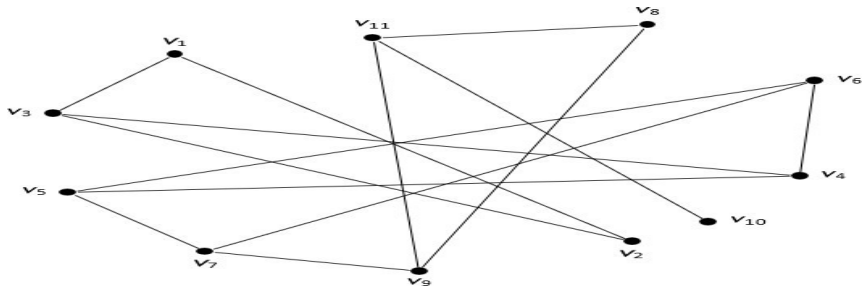
$\Rightarrow DN(G) = 2$

Illustration for odd vertices:

n = 11:



Interval family I



Interval graph G

The dominating sets are $d_1 = \{v_1, v_3, v_5, v_7, v_9, v_{11}\}$

$d_2 = \{v_2, v_4, v_6, v_8, v_{10}\}$

$d_1 \cap d_2 = \{v_1, v_3, v_5, v_7, v_9, v_{11}\} \cap \{v_1, v_3, v_5, v_7, v_9, v_{11}\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_1, v_3, v_5, v_7, v_9, v_{11}\} \cup \{v_1, v_3, v_5, v_7, v_9, v_{11}\}$

$= \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}\}$

$= V(G)$

Therefore $\{d_1, d_2\}$ is a domatic partition

$\Rightarrow DN(G) = 2$

Corollary 1:

Let us consider Fibonacci series 0, 1, 1, 2, 3, 5, 8, 13,……. i.e., $F_n = F_{n-1} + F_{n-2}$; $\forall n \geq 2$ with initial conditions $F_0 = 0$, $F_1 = 1$. By using theorem 1 the domatic number of Fibonacci series $DN(G) = 2$ except 0, 1 .

Proof:

Let us consider the Fibonacci series 0, 1, 1, 2, 3, 5, 8, 13, ……. i.e., $F_n = F_{n-1} + F_{n-2}$; $\forall n \geq 2$ with initial conditions $F_0 = 0$, $F_1 = 1$.

Let $n = 2$ then dominating sets are $d_1 = \{v_1\}$ and $d_2 = \{v_2\}$

Where $d_1 \cap d_2 = \{v_1\} \cap \{v_2\}$

$d_1 \cap d_2 = \emptyset$

And

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$d_1 \cup d_2 = \{v_1\} \cup \{v_2\}$

$d_1 \cup d_2 = \{v_1, v_2\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore $\{d_1, d_2\}$ is a domatic partition.

Therefore domatic number DN(G) = 2

Let n = 3 then the dominating sets are $d_1 = \{v_1\}$ and $d_2 = \{v_2, v_3\}$

Where $d_1 \cap d_2 = \{v_1\} \cap \{v_2, v_3\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_1\} \cup \{v_2, v_3\}$

$d_1 \cup d_2 = \{v_1, v_2, v_3\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore $\{d_1, d_2\}$ is a domatic partition.

Therefore domatic number DN(G) = 2

Let n = 5 then the dominating sets are $d_1 = \{v_2, v_4\}$ and $d_2 = \{v_1, v_3, v_5\}$

Where $d_1 \cap d_2 = \{v_2, v_4\} \cap \{v_1, v_3, v_5\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_2, v_4\} \cup \{v_1, v_3, v_5\}$

$d_1 \cup d_2 = \{v_1, v_2, v_3, v_4, v_5\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore $\{d_1, d_2\}$ is a domatic partition.

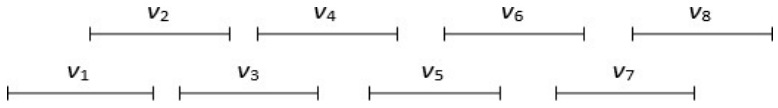
Therefore domatic number DN(G) = 2

Similarly we take n = k, $\forall k \in N$ then the domatic number DN(G) = 2

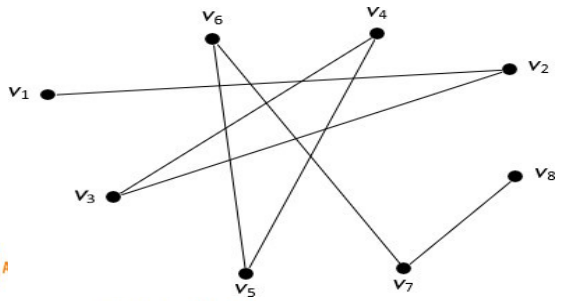
Hence the proof

Illustration:

n = 8:



Interval family I



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Interval graph G

Let n = 8 then the dominating sets are $d_1 = \{v_2, v_4, v_6, v_7\}$ and $d_2 = \{v_1, v_3, v_5, v_8\}$

Where $d_1 \cap d_2 = \{v_2, v_4, v_6, v_7\} \cap \{v_1, v_3, v_5, v_8\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_2, v_4, v_6, v_7\} \cup \{v_1, v_3, v_5, v_8\}$

$d_1 \cup d_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore $\{d_1, d_2\}$ is a domatic partition.

Therefore domatic number DN(G) = 2

Hence the proof

Corollary 2:

Let us consider Arithmetic Progression series 2, 3, 4, 5, ….. i.e., a, a+d, a+2d, …… where a = first term, d = common difference, d = second term - first term. And the geometric series 2, 6, 18, 54, ……. i.e., a, ar, ar^2, ar^3, …… where a = first term

r = common ratio

$$r = \frac{\text{second term}}{\text{first term}}$$

Then the domatic number of A.P is DN(G) = 2 except at a = 1 and the domatic number of G.P is DN(G) = 2

Proof:

Let us consider Arithmetic Progression series 2, 3, 4, 5, ….. i.e., a, a+d, a+2d, …… where a = first term, d = common difference,

d = second term - first term.

d = 3-2

d = 1

Let n = 2 then the dominating sets are $d_1 = \{v_1\}$ and $d_2 = \{v_2\}$

Where $d_1 \cap d_2 = \{v_1\} \cap \{v_2\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_1\} \cup \{v_2\}$

$d_1 \cup d_2 = \{v_1, v_2\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore $\{d_1, d_2\}$ is a domatic partition.

Therefore domatic number DN(G) = 2

Let n = 3 then the dominating sets are $d_1 = \{v_1\}$ and $d_2 = \{v_2, v_3\}$

Where $d_1 \cap d_2 = \{v_1\} \cap \{v_2, v_3\}$

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$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_1\} \cup \{v_2, v_3\}$

$d_1 \cup d_2 = \{v_1, v_2, v_3\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore $\{d_1, d_2\}$ is a domatic partition.

Therefore domatic number DN(G) = 2

Similarly n = k, $\forall k \in N$ then the domatic number DN(G) = 2

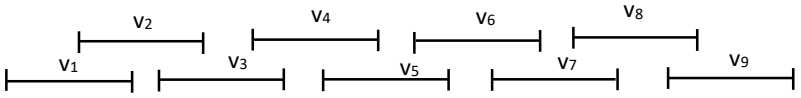
In this similar way we can also prove that the domatic number of geometric series

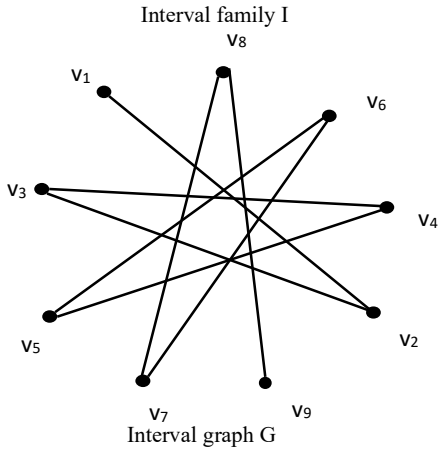
DN(G) = 2

Hence the proof

Illustration:

n = 9:





Let n = 9 then the dominating sets are $d_1 = \{v_2, v_5, v_7, v_9\}$ and $d_2 = \{v_1, v_3, v_4, v_6, v_8\}$

Where $d_1 \cap d_2 = \{v_2, v_5, v_7, v_9\} \cap \{v_1, v_3, v_4, v_6, v_8\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_2, v_5, v_7, v_9\} \cup \{v_1, v_3, v_4, v_6, v_8\}$

$d_1 \cup d_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore $\{d_1, d_2\}$ is a domatic partition.

Therefore domatic number DN(G) = 2

Lemma:

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Let I = {I_1, I_2, …. , I_n} be a finite contains and contained interval family and G be a interval graph corresponding to I. If n = 2 and n = 3 then the domatic number is equal to number of vertices i.e. , DN(G) = n.

Proof:

Let I = {I_1, I_2, …. , I_n} be a finite contains and contained interval family and G be a interval graph corresponding to I.

We know that a collection of partition of G is set of {d_1, d_2, ….. , d_k} is called domatic partition of G if

1) {$d_1, d_2, d_3, \ldots, d_k$} are pair wise disjoint.i.e., $d_1 \cap d_2 \cap d_3 \cap \ldots \cap d_k = \emptyset$
2) $\cup_{i=1}^{k} d_i = V(G)$ i.e., $d_1 \cup d_2 \cup d_3 \cup \ldots \cup d_k = V(G)$
3) Each d_i is a dominating set

By definition the maximum possible number of dominating sets in a domatic partition is domatic number DN(G)

Let n = 2 then the dominating sets are $d_1 = \{v_1\}$ and $d_2 = \{v_2\}$

Where $d_1 \cap d_2 = \{v_1\} \cap \{v_2\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_1\} \cup \{v_2\}$

$d_1 \cup d_2 = \{v_1, v_2\}$

$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore {d_1, d_2} is a domatic partition.

Therefore domatic number DN(G) = 2 = number of vertices

Let n = 3 then the dominating sets are $d_1 = \{v_1\}$ and $d_2 = \{v_2\}$, $d_3 = \{v_3\}$

Where $d_1 \cap d_2 \cap d_3 = \{v_1\} \cap \{v_2\} \cap \{v_3\}$

$d_1 \cap d_2 \cap d_3 = \emptyset$

And

$d_1 \cup d_2 \cup d_3 = \{v_1\} \cup \{v_2\} \cup \{v_3\}$

$d_1 \cup d_2 \cup d_3 = V(G)$

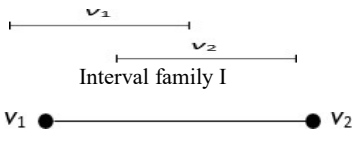
And d_1, d_2, d_3 itself is a dominating sets. Therefore {d_1, d_2, d_3} is a domatic partition.

Therefore domatic number DN(G) = 3 = number of vertices.

Hence the proof.

Illustration:

n = 2:



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Interval graph G

Let n = 2 then the dominating sets are $d_1 = \{v_1\}$ and $d_2 = \{v_2\}$

Where $d_1 \cap d_2 = \{v_1\} \cap \{v_2\}$

$d_1 \cap d_2 = \emptyset$

And

$d_1 \cup d_2 = \{v_1\} \cup \{v_2\}$

$d_1 \cup d_2 = \{v_1, v_2\}$

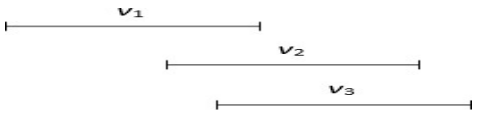
$d_1 \cup d_2 = V(G)$

And d_1, d_2 itself is a dominating sets. Therefore $\{d_1, d_2\}$ is a domatic partition.

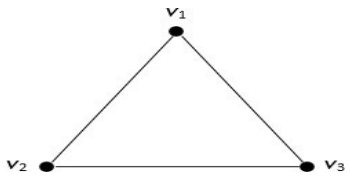
Therefore domatic number DN(G) = 2 = number of vertices.

Illustration:

n = 3:



Interval family I



Interval graph G

Let n = 3 then the dominating sets of G are $d_1 = \{v_1\}$ and $d_2 = \{v_2\}$, $d_3 = \{v_3\}$

Where $d_1 \cap d_2 \cap d_3 = \{v_1\} \cap \{v_2\} \cap \{v_3\}$

$d_1 \cap d_2 \cap d_3 = \emptyset$

And

$d_1 \cup d_2 \cup d_3 = \{v_1\} \cup \{v_2\} \cup \{v_3\}$

$d_1 \cup d_2 \cup d_3 = V(G)$

And d_1, d_2, d_3 itself is a dominating sets. Therefore $\{d_1, d_2, d_3\}$ is a domatic partition.

Therefore domatic number DN(G) = 3 = number of vertices.

Conclusion: In this paper, we determine the domatic number of an interval graph corresponding to an interval family. A family of intervals is primarily used in mathematics and computer Science to model and analyze interval graphs. Interval graphs provide a natural mathematical model for networks and systems in which relationships, interactions, or coverage occurs along a linear domain. The domination and domatic parameters can be used to study coverage, redundancy, and fault tolerance in such networks. The concept of a

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domatic number has useful in real-life interpretations such as wireless sensor networks, computer and communication networks, Emergency service placement, military (or) security surveillance, social networks and transportation networks etc.

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